

HyperPCTL: A Temporal Logic for Probabilistic Hyperproperties

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Presentation outline

- 1 Motivation
- 2 HyperPCTL Syntax and Semantics
- 3 HyperPCTL in Action
- 4 HyperPCTL Model Checking
- 5 Conclusion

Motivation

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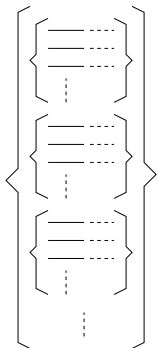
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Classical trace properties cannot express relation among multiple traces

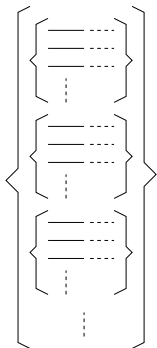
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A **hyperproperty** is a set of sets of traces.



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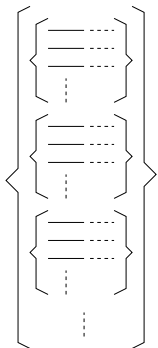


Information-flow security:

- Noninterference
- Observational determinism
- Declassification
- Noninference

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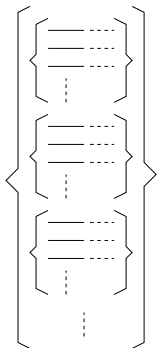
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Consistency models (concurrency):

- Linearizability
- Eventual/causal consistency

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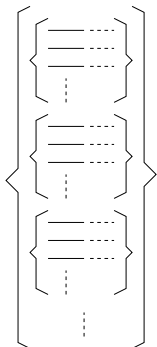
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Temporal logics for hyperproperties:

- HyperLTL
- HyperCTL*

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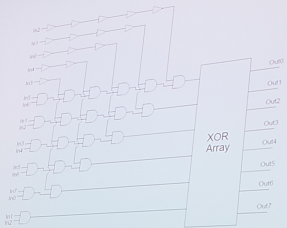
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Hyperproperty Satisfaction

A system P **satisfies** a hyperproperty ψ (denoted, $P \models \psi$) iff $\text{Traces}(P) \in \psi$; i.e., language equality.

Timed Hyperproperties

Countermeasure to Attack



FSA attack resilient ckt: All input-to-output paths have same delays
Manually hand-crafted solution [Schaumont et al, DATE 2014]

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- If $h = 5$, then at termination, $\mathbb{P}(l = 1) = 1/4096$ and $\mathbb{P}(l = 2) = 4095/4096$.

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$$\forall \sigma. \forall \sigma'. \left(\text{init}_\sigma \wedge \text{init}_{\sigma'} \wedge h_\sigma \neq h_{\sigma'} \right) \Rightarrow$$

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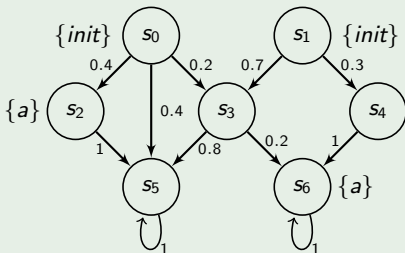
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HyperPCTL Semantics

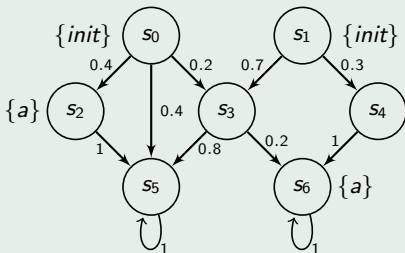
Example



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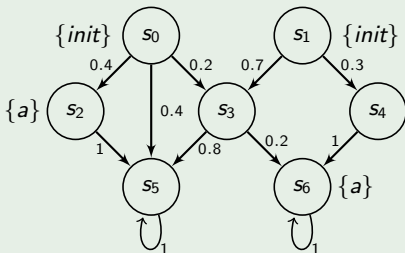


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$$Pr[\mathcal{A}(D_1) \in S] \leq e^\epsilon \cdot Pr[\mathcal{A}(D_2) \in S].$$

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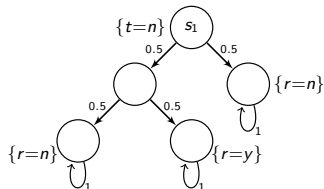
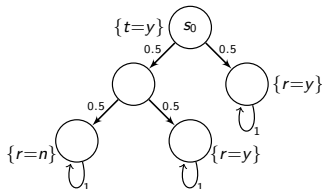
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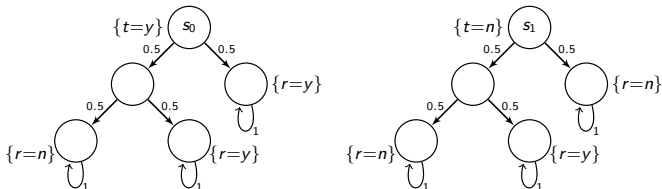


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HyperPCTL formula for DP

$$\forall \sigma. \forall \sigma'. \left[\left((t=n)_\sigma \wedge (t=y)_{\sigma'} \right) \Rightarrow \left(\mathbb{P} \left(\diamond (r=n)_\sigma \right) \leq e^{\ln 3} \cdot \mathbb{P} \left(\diamond (r=n)_{\sigma'} \right) \right) \right] \wedge$$

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$$\psi_{\text{pc}_1} = \forall \sigma. \forall \sigma'. c_\sigma \wedge \left(\mathbb{P}(\diamond e_\sigma) > \mathbb{P}(\neg c_{\sigma'} \cup e_{\sigma'}) \right).$$

HyperPCTL Examples

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Probabilistic Bisimulation

$$\varphi_{\text{pb}} = \forall \sigma. \forall \sigma'. \bigwedge_{i=1}^k \left[(a_{\sigma}^i \wedge a_{\sigma'}^i) \Rightarrow \left[\psi^{AP} \wedge \bigwedge_{j=1}^k \mathbb{P}(\bigcirc a_{\sigma}^j) = \mathbb{P}(\bigcirc a_{\sigma'}^j) \right] \right]$$

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The HyperPCTL model checking problem is **PSPACE-hard** in the number of quantifiers in the formula.

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- Differential privacy

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We introduced a temporal logic to express **probabilistic hyperproperties**.

HyperPCTL extends PCTL by allowing explicit and simultaneous quantification over initial states of a discrete-time Markov chain.

We showed that HyperPCTL can express interesting requirements:

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- Probabilistic noninterference
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We presented a **polynomial-time** model checking algorithm in the size of the input DTMC (**exponential** in the size of the input HyperPCTL formula).

Future Work

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On-the-fly model checking algorithm without full blown generation of the self-composition.

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DTMC **repair** for HyperPCTL.

Thank you!